STRAIGHT

LINE

(KEY CONCEPTS + SOLVED EXAMPLES)

STRAIGHT LINE

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KEY CONCEPTS

1. Equation of Straight Line

A relation between x and y which is satisfied by co-ordinates of every point lying on a line is called the equation of Straight Line. Every linear equation in two variable x and y always represents a straight line.

eg. $3x + 4y = 5$, $-4x + 9y = 3$ etc.

General form of straight line is given by

$ax + by + c = 0$.

2. Equation of Straight line Parallel to Axes

(i) Equation of x axis $\Rightarrow y = 0$.

Equation a line parallel to x axis (or perpendicular to y-axis) at a distance 'a' from it $\Rightarrow y = a$.

(ii) Equation of y axis $\Rightarrow x = 0$.

Equation of a line parallel to y-axis (or perpendicular to x axis) at a distance 'a' from it $\Rightarrow x = a$.

eg. Equation of a line which is parallel to x-axis and at a distance of 4 units in the negative direction is $y = -4$.

3. Slope of a Line

If θ is the angle made by a line with the positive direction of x axis in anticlockwise sense, then the value of $\tan\theta$ is called the Slope (also called gradient) of the line and is denoted by m or slope $\Rightarrow m = \tan\theta$

eg. A line which is making an angle of 45º with the x-axis then its slope is $m = \tan 45^\circ = 1$.

Note :

(i) Slope of x axis or a line parallel to x-axis is $\tan 0^\circ = 0$.

(ii) Slope of y axis or a line parallel to y-axis is $\tan 90^\circ = \infty$.

(iii) The slope of a line joining two points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$.

eg. Slope of a line joining two points (3, 5) and (7, 9) is $= \frac{9-5}{7-3} = \frac{4}{4} = 1$.

4. Different forms of the Equation of Straight line

4.1 Slope - Intercept Form :

The equation of a line with slope m and making an intercept c on y–axis is $y = mx + c$. If the line passes through the origin, then $c = 0$. Thus the equation of a line with slope m and passing through the origin $y = mx$.

4.2 Slope Point Form :

The equation of a line with slope m and passing through a point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

4.3 Two Point Form :

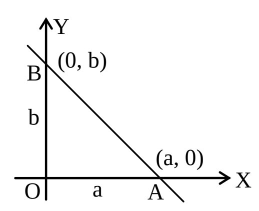
The equation of a line passing through two given points (x_1, y_1) and (x_2, y_2) is -

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

4.4 Intercept Form :

The equation of a line which makes intercept a and b on the x-axis and y-axis respectively is $\frac{x}{a} + \frac{y}{b} = 1$.

Here, the length of intercept between the co-ordinates axis $= \sqrt{a^2 + b^2}$



Area of $\Delta OAB = \frac{1}{2}$ OA. OB $= \frac{1}{2}$ a.b.

4.5 Normal (Perpendicular) Form of a Line :

If p is the length of perpendicular on a line from the origin and α is the inclination of perpendicular with x- axis then equation on this line is

$$x\cos\alpha + y\sin\alpha = p$$

4.6 Parametric Form (Distance Form) :

If θ be the angle made by a straight line with x-axis which is passing through the point (x_1, y_1) and r be the distance of any point (x, y) on the line from the point (x_1, y_1) then its equation.

$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$$

5. Reduction of general form of Equations into Standard forms

General Form of equation ax + by + c = 0 then its-

(i) Slope Intercept Form is

$y = -\frac{a}{b}x - \frac{c}{b}$, here slope $m = -\frac{a}{b}$, Intercept

$C = \frac{c}{b}$

(ii) Intercept Form is

$\frac{x}{-c/a} + \frac{y}{-c/b} = 1$, here x intercept is

$= -c/a$, y intercept is $= -c/b$

(iii) Normal Form is to change the general form of a line into normal form, first take c to right hand side and make it positive, then divide the whole equation by $\sqrt{a^2 + b^2}$ like

$$-\frac{ax}{\sqrt{a^2+b^2}} - \frac{by}{\sqrt{a^2+b^2}} = \frac{c}{\sqrt{a^2+b^2}},$$

here $\cos\alpha = \frac{a}{\sqrt{a^2+b^2}}$, $\sin\alpha = \frac{b}{\sqrt{a^2+b^2}}$ and

$$p = \frac{c}{\sqrt{a^2+b^2}}$$

6. Position of a point relative to a line

(i) The point (x_1, y_1) lies on the line ax + by + c = 0 if, $ax_1 + by_1 + c = 0$

(ii) If $P(x_1, y_1)$ and $Q(x_2, y_2)$ do not lie on the line ax + by + c = 0 then they are on the same side of the line, if $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ are of the same sign and they lie on the opposite sides of line if $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ are of the opposite sign.

(iii) (x_1, y_1) is on origin or non origin sides of the line ax + by + c = 0 if $ax_1 + by_1 + c = 0$ and c are of the same or opposite signs.

7. Angle between two Straight lines

The angle between two straight lines $y = m_1x + c_1$ and $y = m_2x + c_2$ is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$$

Note :

(i) If any one line is parallel to y axis then the angle between two straight line is given by

$$\tan\theta = \pm\frac{1}{m}$$

Where m is the slope of other straight line

(ii) If the equation of lines are $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ then above formula would be

$$\tan\theta = \left|\frac{a_1b_2 - b_1a_2}{a_1a_2 + b_1b_2}\right|$$

(iii) Here two angles between two lines, but generally we consider the acute angle as the angle between them, so in all the above formula we take only positive value of $\tan\theta$.

7.1 Parallel Lines :

Two lines are parallel, then angle between them is 0

$$\Rightarrow \frac{m_1 - m_2}{1 + m_1m_2} = \tan 0^\circ = 0$$

$$\Rightarrow m_1 = m_2$$

Note : Lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel $\Leftrightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$

7.2 Perpendicular Lines :

Two lines are perpendicular, then angle between them is 90º

$$\Rightarrow \frac{m_1 - m_2}{1 + m_1m_2} = \tan 90^\circ = \infty$$

$$\Rightarrow m_1m_2 = -1$$

Note : Lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are perpendicular then $a_1a_2 + b_1b_2 = 0$

7.3 Coincident Lines :

Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are coincident only and only if $\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

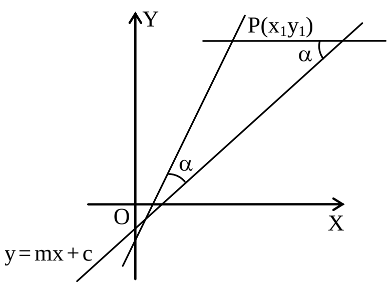
8. Equation of Parallel & Perpendicular lines

(i) Equation of a line which is parallel to $ax + by + c = 0$ is $ax + by + k = 0$

(ii) Equation of a line which is perpendicular to $ax + by + c = 0$ is $bx - ay + k = 0$

The value of k in both cases is obtained with the help of additional information given in the problem.

9. Equation of Straight lines through (X_1, Y_1) making an angle a with = mx + c



$$y - y_1 = \frac{m \mp \tan\alpha}{1 \pm m\tan\alpha}(x - x_1)$$

10. Length of Perpendicular

The length P of the perpendicular from the point (x_1, y_1) on the line $ax + by + c = 0$ is given by

$$P = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Note :

(i) Length of perpendicular from origin on the line $ax + by + c = 0$ is $c / \sqrt{a^2 + b^2}$

(ii) Length of perpendicular from the point (x_1, y_1) on the line $x \cos\alpha + y \sin\alpha = p$ is -

$x_1\cos\alpha + y_1 \sin\alpha = p$

10.1 Distance between Two Parallel Lines :

The distance between two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is

$$\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

Note :

(i) Distance between two parallel lines $ax + by + c_1 = 0$ and $kax + kby + c_2 = 0$ is

$$\frac{\left|c_1 - \frac{c_2}{k}\right|}{\sqrt{a^2 + b^2}}$$

(ii) Distance between two non parallel lines is always zero.

11. Condition of Concurrency

Three lines $a_1x + b_1y + c_1= 0$, $a_2x + b_2y + c_2= 0$ and $a_3x + b_3y + c_3 = 0$ are said to be concurrent, if they passes through a same point. The condition for their concurrency is

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

Again, to test the concurrency of three lines, first find out the point of intersection of any two of the three lines. If this point lies on the remaining lines then the three lines are concurrent.

Note : If P = 0, Q = 0, R = 0 the equation of any three line and P + Q + R = 0 the line are concurrent. But its converse is not true i.e. if the line are concurrent then it is not necessary that P + Q + R = 0

12. Bisector of Angle between two Straight line

(i) Equation of the bisector of angles between the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

(ii) To discriminate between the acute angle bisector and the obtuse angle bisector : If θ be the angle between one of the lines and one of the bisector, find $\tan\theta$. If $|\tan\theta| < 1$ then $2\theta < 90^{o}$ so that this bisector is the acute angle bisector, If $|\tan\theta| > 1$, then we get the bisector to be the obtuse angle bisector.

(iii) First write the equation of the lines so that the constant terms are positive. Then

(a) If $a_1a_2 + b_1b_2 > 0$ then on taking positive sign in the above bisectors equation we shall get the obtuse angle bisector and on taking negative sign we shall get the acute angle bisector.

(b) If $a_1a_2 + b_1b_2 < 0$, the positive sign give the acute angle and negative sign gives the obtuse angle bisector.

(c) On taking positive sign we shall get equation of the bisector of the angle which contains the origin and negative sign gives the equation of the bisector which does not contain origin.

Note : This is also the bisector of the angle in which origin lies (since c_1, c_2 are positive and it has been obtained by taking positive sign)

13. Lines passing through the point of intersection of two lines

If equation of two lines $P = a_1x + b_1y + c_1 = 0$ and $Q = a_2x + b_2y + c_2 = 0$, then the equation of the lines passing through the point of intersection of these lines is $P + \lambda Q = 0$ or $(a_1x + b_1y + c = 0) + \lambda(a_2x + b_2y + c_2 = 0) = 0$; Value of λ is obtained with the help of the additional information given in the problem.

SOLVED EXAMPLES

Ex.1 The equation of the line which passes through the point (3, 4) and the sum of its intercept on the axes is 14, is -

(A) $4x - 3y = 24,\ x - y = 7$

(B) $4x + 3y = 24,\ x + y = 7$

(C) $4x + 3y + 24 = 0,\ x + y + 7 = 0$

(D) $4x - 3y + 24 = 0,\ x - y + 7 = 0$

Sol. Let the equation of the line be $\frac{x}{a} + \frac{y}{b} = 1$ ...(1)

This passes through (3, 4), therefore

$$\frac{3}{a} + \frac{4}{b} = 1 \qquad ...(2)$$

It is given that $a + b = 14 \Rightarrow b = 14 - a$. Putting $b = 14 - a$ in (2), we get

$$\frac{3}{a} + \frac{4}{b} = 1 \quad \Rightarrow a^2 - 13a + 42 = 0$$

$\Rightarrow (a - 7)(a - 6) = 0 \Rightarrow a = 7, 6$

For $a = 7$, $b = 14 - 7 = 7$ and for $a = 6$, $b = 14 - 6 = 8$.

Putting the values of a and b in (1), we get the equations of the lines

$$\frac{x}{7} + \frac{y}{7} = 1 \quad \text{and} \quad \frac{x}{6} + \frac{y}{8} = 1$$

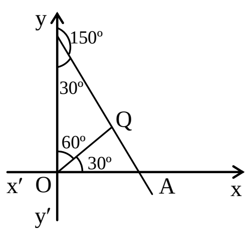
or $x + y = 7$ and $4x + 3y = 24$ **Ans. [B]**

Ex.2 The length of the perpendicular from the origin to a line is 7 and the line makes an angle of 150º with the positive direction of y-axis. The equation of the line is –

(A) $\sqrt{3}\,x + y = 14$ (B) $\sqrt{3}\,x - y = 14$

(C) $\sqrt{3}\,x + y + 14 = 0$ (D) $\sqrt{3}\,x - y + 14 = 0$

Sol. Here $p = 7$ and $\alpha = 30º$



$\therefore$ Equation of the required line is

$x \cos 30º + y \sin 30º = 7$

or $x\frac{\sqrt{3}}{2} + y \times \frac{1}{2} = 7$

or $\sqrt{3}\,x + y = 14$ **Ans. [A]**

Ex.3 If the intercept made by the line between the axes is bisected at the point (x_1, y_1), then its equation is -

(A) $\frac{x}{x_1} + \frac{y}{y_1} = 2$ (B) $\frac{x}{x_1} + \frac{y}{y_1} = 1$

(C) $\frac{x}{x_1} + \frac{y}{y_1} = \frac{1}{2}$ (D) None of these

Sol. Let the equations of the line be $\frac{x}{a} + \frac{y}{b} = 1$, then the coordinates of point of intersection of this line and x-axis and y-axis are respectively (a, 0). (0, b). Hence mid point of the intercept is (a/2, b/2).

$\therefore$ $a/2 = x_1 \Rightarrow a = 2x_1$ and $b/2 = y_1$

$\Rightarrow b = 2y_1$

Hence required equation of the line is

$$\frac{x}{2x_1} + \frac{y}{2y_1} = 1$$

$$\Rightarrow \frac{x}{x_1} + \frac{y}{y_1} = 2$$ **Ans. [A]**

Ex.4 The distance of the point (2, 3) from the line $2x - 3y + 9 = 0$ measured along a line $x - y + 1 = 0$, is -

(A) $\sqrt{2}$ (B) $4\sqrt{2}$

(C) $\sqrt{8}$ (D) $3\sqrt{2}$

Sol. The slope of the line $x - y + 1 = 0$ is 1. So it makes an angle of 45º with x-axis.

The equation of a line passing through (2, 3) and making an angle of 45º is

$$\frac{x-2}{\cos 45º} = \frac{y-3}{\sin 45º} = r$$

$$\left[\text{Using } \frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r\right]$$

co-ordinats of any point on this line are

$(2 + r\cos 45º,\ 3 + r \sin 45º)$ or $\left(2 + \frac{r}{\sqrt{2}}, 3 + \frac{r}{\sqrt{2}}\right)$

If this point lies on the line $2x - 3y + 9 = 0$,

then $4 + r\sqrt{2} - 9 - \frac{3r}{\sqrt{2}} + 9 = 0$

$\Rightarrow r = 4\sqrt{2}$.

So the required distance $= 4\sqrt{2}$. **Ans. [B]**

Ex.5 If x + 2y = 3 is a line and A(–1, 3); B(2, –3); C(4, 9) are three points, then -

(A) A is on one side and B, C are on other side of the line

(B) A, B are on one side and C is on other side of the line

(C) A, C on one side and B is no other side of the line

(D) All three points are on one side of the line

Sol. Substituting the coordinates of points A, B and C in the expression x + 2y – 3, we get

The value of expression for A is

= –1 + 6 – 3 = 2 > 0

The value of expression for B is

= 2 – 6 – 3 = – 7 < 0

The value of expression for C is

= 4 + 18 – 3 = 19 > 0

$\because$ Signs of expressions for A, C are same while for B, the sign of expression is different

$\therefore$ A, C are on one side and B is on other side of the line

Ans. [C]

Ex.6 The equation of two equal sides of an isosceles triangle are 7x – y + 3 = 0 and x + y – 3 = 0 and its third side is passes through the point (1, –10). The equation of the third side is

(A) x – 3y – 31 = 0 but not 3x + y + 7 = 0

(B) neither 3x + y + 7 = 0 nor x – 3y – 31 = 0

(C) 3x = y + 7 = 0 or x – 3y – 31 = 0

(D) 3x + y + 7 = 0 but not x – 3y – 31 = 0

Sol. Third side passes through (1, –10) so let its equation be y + 10 = m(x –1)

If it makes equal angle, say θ with given two sides, then

$$\tan\theta = \frac{m-7}{1+7m} = \frac{m-(-1)}{1+m(-1)} \Rightarrow m = -3 \text{ or } 1/3$$

Hence possible equations of third side are

$$y + 10 = -3(x-1) \text{ and } y + 10 = \frac{1}{3}(x-1)$$

or 3x + y + 7 = 0 and x – 3y – 31 = 0

Ans.[C]

Ex.7 Triangle formed by lines x + y = 0, 3x + y = 4 and x + 3y = 4 is -

(A) equilateral (B) right angled

(C) isosceles (D) None of these

Sol. Slope of the given lines are –1, –3, $-\frac{1}{3}$ respectively

Let $m_1 = -\frac{1}{3}$, $m_2 = -1$, $m_3 = -3$

$$\therefore \tan A = \frac{-\frac{1}{3}+1}{1+\frac{1}{3}.1} \Rightarrow A = \tan^{-1}\left(\frac{1}{2}\right)$$

$$\tan B = \frac{-1+3}{1+1.3} \Rightarrow B = \tan^{-1}\left(\frac{1}{2}\right)$$

$$\text{and } \tan C = \frac{-\frac{1}{3}+1}{1+3.\frac{1}{3}} \Rightarrow C = \tan^{-1}\left(-\frac{4}{3}\right)$$

$\because$ ∠A = ∠B, Hence triangle is isosceles triangle.

Ans.[C]

Ex.8 If A(–2,1), B(2,3) and C(–2,–4) are three points, then the angle between BA and BC is -

(A) $\tan^{-1}\left(\frac{3}{2}\right)$ (B) $\tan^{-1}\left(\frac{2}{3}\right)$

(C) $\tan^{-1}\left(\frac{7}{4}\right)$ (D) None of these

Sol. Let m_1 and m_2 be the slopes of BA and BC respectively. Then

$$m_1 = \frac{3-1}{2-(-2)} = \frac{2}{4} = \frac{1}{2} \quad \text{and} \quad m_2 = \frac{-4-3}{-2-2} = \frac{7}{4}$$

Let θ be the angle between BA and BC. Then

$$\tan\theta = \left|\frac{m_2-m_1}{1+m_1m_2}\right| = \left|\frac{\frac{7}{4}-\frac{1}{2}}{1+\frac{7}{4}\times\frac{1}{2}}\right| = \left|\frac{\frac{10}{8}}{\frac{15}{8}}\right| = \pm\frac{2}{3}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{2}{3}\right)$$

Ans. [B]

Ex.9 The area of the parallelogram formed by the lines 4y – 3x = 1, 4y – 3x – 3 = 0, 3y – 4x + 1 = 0, 3y – 4x + 2 = 0 is -

(A) 3/8 (B) 2/7

(C) 1/6 (D) None of these

Sol. Let the equation of sides AB, BC, CD and DA of parallelogram ABCD are respectively

$y = \frac{3}{4}x + \frac{1}{4}$ ...(1); $y = \frac{3}{4}x + \frac{3}{4}$ ...(2)

$y = \frac{4}{3}x - \frac{1}{3}$ ...(3); $y = \frac{4}{3}x - \frac{2}{3}$ ...(4)

Here $m = \frac{3}{4}$, $n = \frac{4}{3}$, $a = \frac{1}{4}$, $b = \frac{3}{4}$,

$c = -\frac{1}{3}$, $d = -\frac{2}{3}$

$\therefore$ Area of parallelogram ABCD

$$= \left|\frac{(a-b)(c-d)}{m-n}\right| = \left|\frac{\left(\frac{1}{4}-\frac{3}{4}\right)\left(-\frac{1}{3}+\frac{2}{3}\right)}{\frac{3}{4}-\frac{4}{3}}\right|$$

$$= \left|\frac{-\frac{1}{2}\times\frac{1}{3}}{-\frac{7}{12}}\right| = \frac{2}{7}$$

Ans. [B]

Ex.10 The equation of a line parallel to $ax + by + c' = 0$ and passing through the point (c, d) is -

(A) a(x + c) – b(y + d) = 0

(B) a(x + c) + b(y + d) = 0

(C) a(x – c) + b(y – d) = 0

(D) None of these

Sol. Equation of a line parallel to ax + by + c = 0 is written as

ax + by + k = 0 ...(1)

f it passes through (c, d), then

ac + bd + k = 0 ...(2)

Subtracting (2) and (1), we get

a(x – c) + b(y – d) = 0

Which is the required equation of the line.

Ans.[C]

Ex.11 A straight line L perpendicular to the line 5x – y = 1. The area of the triangle formed by the line L and co-ordinates axes is 5, then the equation of line, is -

(A) x + 5y = ± 5 (B) $x + 5y = \pm\sqrt{2}$

(C) $x + 5y = \pm 5\sqrt{2}$ (D) None of these

Sol. Let the line L cut the axes at A and B say. OA = a, OB = b

$\therefore$ Area Δ OAB $= \frac{1}{2}ab = 5$ ...(1)

Now equation of line perpendicular to lines 5x – y = 1 is x + 5y = k

Putting x = 0, y = = b, y = 0, x = k = a

$\therefore \frac{1}{2}k. k/5 = 5$ from ... (1)

$k^2 = 50 \Rightarrow k = 5\sqrt{2}$

Hence the required line is $x + 5y = \pm 5\sqrt{2}$

Ans.[C]

Note : Trace the line approximately and try to make use of given material as per the question.

Ex.12 The sides AB, BC, CD and DA of a quadrilateral have the equations x + 2y = 3, x = 1, x – 3y = 4, 5x + y + 12 = 0 respectively, then the angle between the diagonals AC and BD is -

(A) 60º (B) 45º

(C) 90º (D) None of these

Sol. Solving for A,

x + 2y – 3 = 0

5x + y + 12 = 0

$$\Rightarrow \frac{x}{+24+3} = \frac{y}{-15-12} = \frac{1}{-9}$$

$\therefore$ A (–3, 3)

Similarly B(1,1), C(1, –1), D(–2, –2)

Now m_1 = slope of AC = – 1

m_2 = slope of BD = 1

$m_1m_2 = -1$ $\therefore$ the angle required is 90º

Ans. [C]

Ex.13 If the lines ax + by + c = 0, bx + cy + a = 0 and cx + ay + b = 0 are concurrent, then -

(A) a – b – c = 0 (B) a + b + c = 0

(C) b + c – a = 0 (D) a + b – c = 0

Sol. If the lines are concurrent, then $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$

$\Rightarrow 3abc - a^3 - b^3 - c^3 = 0$

$\Rightarrow (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

$\Rightarrow (a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2] = 0$

$\Rightarrow a + b + c = 0$

$[\therefore (a - b)^2 + (b - c)^2 + (c - a)^2 \neq 0]$ **Ans. [B]**

Ex.14 The vertices of ΔOBC are respectively (0, 0), (–3, –1) and (–1, –3). The equation of line parallel to BC and at a distance 1/2 from O which intersects OB and OC is -

(A) $2x + 2y + \sqrt{2} = 0$ (B) $2x - 2y + \sqrt{2} = 0$

(C) $2x + 2y - \sqrt{2} = 0$ (D) None of these

Sol. Slope of BC $= \frac{-3+1}{-1+3} = -1$

Now equation of line parallel to BC is

$y = -x + k \Rightarrow y + x = k$

Now length of perpendicular from O on this line

$= \pm \frac{k}{\sqrt{2}} = \frac{1}{2} \Rightarrow k = -\frac{\sqrt{2}}{2}$

$\therefore$ Equation of required line is

$2x + 2y + \sqrt{2} = 0$ **Ans. [A]**

Ex.15 The equation of a line through the point of intersection of the lines $x - 3y + 1 = 0$ and $2x + 5y - 9 = 0$ and whose distance from the origin is $\sqrt{5}$, is -

(A) $2x + y - 5 = 0$ (B) $2x - y + 5 = 0$

(C) $2x + y - 10 = 0$ (D) $2x - y - 10 = 0$

Sol. Let the required line by method $P + \lambda Q = 0$ be

$(x - 3y + 1) + \lambda(2x + 5y - 9) = 0$

$\therefore$ perpendicular from (0, 0) $= \sqrt{5}$ gives

$$\frac{1-9\lambda}{\sqrt{(1-2\lambda)^2 + (5-3\lambda)^2}} = \sqrt{5},$$

squaring and simplifying $(8\lambda - 7)^2 = 0$

$\Rightarrow \lambda = 7/8$

Hence the line required is

$(x - 3y + 1) + 7/8\,(2x + 5y - 9) = 0$

or $22x + 11y - 55 = 0 \Rightarrow 2x + y - 5 = 0$

Ans.[A]

Note: Here to find the point of intersection is not necessary.

Ex.16 A variable line passes through the fixed point P. If the algebraic sum of perpendicular distances of the points (2, 0); (0, 2) and (1, 1) from the line is zero, then P is -

(A) (1,1) (B) (1, –1)

(C) (2, 2) (D) None of these

Sol. Let equation of variable line is

$ax + by + c = 0$ ...(1)

Now sum of perpendicular distance

$$\frac{2a+c}{\sqrt{a^2+b^2}} + \frac{2b+c}{\sqrt{a^2+b^2}} + \frac{a+b+c}{\sqrt{a^2+b^2}} = 0$$

$\Rightarrow a + b + c = 0$ ... (2)

on subtracting (2) from (1), we get

$a(x - 1) + b(y - 1) = 0$

Which obviously passes through a fixed point P(1, 1). **Ans. [A]**

Ex.17 The bisector of the acute angle between the lines $3x - 4y + 7 = 0$ and $12x + 5y - 2 = 0$, is

(A) $11x + 3y - 9 = 0$

(B) $21x + 77y - 101 = 0$

(C) $11x - 3y + 9 = 0$

(D) None of these

Sol. Here equation of bisectors

$$\frac{3x-4y+7}{5} = \pm \frac{12x+5y-2}{13}$$

Which give, $11x - 3y + 9 = 0$ and

$21x + 77y - 101 = 0$

Now angle between the line $3x - 4y + 7 = 0$ and one bisector $11x - 3y + 9 = 0$ is

$$|\tan\theta| = \left|\frac{-9+44}{33+12}\right| = \left|\frac{35}{45}\right| < 1$$

Hence the bisector is the required.

$11x - 3y + 9 = 0$ **Ans.[C]**

Ex.18 The equation of two straight lines through (7, 9) and making an angle of 60º with the line $x - \sqrt{3}\,y - 2\sqrt{3} = 0$ is -

(A) $x = 7,\ x + \sqrt{3}\,y = 7 + 9\sqrt{3}$

(B) $x = \sqrt{3},\ x + \sqrt{3}\,y = 7 + 9\sqrt{3}$

(C) $x = 7,\ x - \sqrt{3}\,y = 7 + 9\sqrt{3}$

(D) $x = \sqrt{3},\ x - \sqrt{3}\,y = 7 + 9\sqrt{3}$

Sol. We know that the equations of two straight lines which pass through a point (x_1, y_1) and make a given angle α with the given straight line $y = mx + c$ are

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here $x_1 = 7$, $y_1 = 9$, $\alpha = 60º$ and m = slope of the line $x - \sqrt{3}\,y - 2\sqrt{3} = 0$

So, $m = \frac{1}{\sqrt{3}}$

So, the equation of the required lines are

$$y - 9 = \frac{\frac{1}{\sqrt{3}} + \tan 60^o}{1 - \frac{1}{\sqrt{3}}\tan 60^o}(x - 7)$$

and $$y - 9 = \frac{\frac{1}{\sqrt{3}} - \tan 60^o}{1 + \frac{1}{\sqrt{3}}\tan 60^o}(x - 7)$$

or $$(y - 9)\left(1 - \frac{1}{\sqrt{3}}\tan 60^o\right) = \left(\frac{1}{\sqrt{3}} + \tan 60^o\right)(x - 7)$$

and $$(y - 9)\left(1 + \frac{1}{\sqrt{3}}\tan 60^o\right) = \left(\frac{1}{\sqrt{3}} - \tan 60^o\right)(x - 7)$$

or $0 = \left(\frac{1}{\sqrt{3}} + \sqrt{3}\right)(x - 7) \Rightarrow x - 7 = 0$

and $(y - 9)2 = \left(\frac{1}{\sqrt{3}} - \sqrt{3}\right)(x - 7) \Rightarrow x + \sqrt{3}\,y = 7 + 9\sqrt{3}$

Hence the required lines are $x = 7$ and $x + \sqrt{3}\,y = 7 + 9\sqrt{3}$ **Ans. [A]**

Ex.19 If the lines $x + 2ay + a = 0$, $x + 3by + b = 0$ and $x + 4cy + c = 0$ are concurrent, then a, b and c are in

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

Sol. Given lines will be concurrent if

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0 \Rightarrow -bc + 2ac - ab = 0$$

$\Rightarrow b = \frac{2ac}{a + c}$

$\Rightarrow$ a,b,c are in H.P. **Ans.[C]**

Ex.20 If the sides of triangle are $x + y - 5 = 0$, $x - y + 1 = 0$ and $y - 1 = 0$, then its circumcentre is -

(A) (2, 1) (B) (2, –2)

(C) (1, 2) (D) (1, –2)

Sol. Here the sides $x + y - 5 = 0$ and $x - y + 1$ are perpendicular to each other, therefore $y = 1$ will be hypotenuse of the triangle. Now its middle point will be the circumcentre.

Now solving the pair of equations

$x + y - 5 = 0$, $y - 1 = 0$

and $x - y + 1 = 0$, $y - 1 = 0$, we get

$P \equiv (4, 1)$, $Q \equiv (0, 1)$

Mid point of PQ or circumcentre = (2, 1)

Ans. [A]

Ex.21 If P_1 and P_2 be perpendicular from the origin upon the straight lines $x\sec\theta + y\operatorname{cosec}\theta = a$ and $x\cos\theta - y\sin\theta = a\cos 2\theta$ respectively, then the value of $4P_1^2 + P_2^2$ is -

(A) a^2 (B) $2a^2$

(C) $3a^2$ (D) $4a^2$

Sol. We have P_1 = length of perpendicular from (0, 0) on $x \sec\theta + y \operatorname{cosec}\theta = a$

i.e. $P_1 = \frac{a}{\sqrt{\sec^2\theta + \operatorname{cosec}^2\alpha}} = a \sin\theta \cos\theta$

$= \frac{a}{2}\sin 2\theta$ or $2P_1 = a \sin 2\theta$

P_2 = Length of the perpendicular from (0, 0) on

$x \cos\theta - y \sin\theta = a \cos 2\theta$

$P_2 = \frac{a\cos 2\theta}{\sqrt{\cos^2\theta + \sin^2\theta}} = a \cos 2\theta$

$4P_1^2 + P_2^2 = a^2 \sin^2 2\theta + a^2 \cos^2 2\theta = a^2$

Ans.[A]